Oversubscription Dimensioning of Next-Generation PONs with Different Service Levels

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***Abstract*—This letter provides a methodology for planning Passive Optical Networks using the oversubscription concept and shows its applicability in the dimensioning of 1 Gb/s access to both basic (i.e. residential) and premium (i.e. business) users with different service level requirements and activity patterns. We show that only Next-Generation PON networks can reach a large number of users with acceptable service levels.**

***Index Terms*—Next-Generation Optical Access (NGOA); Network Dimensioning; Oversubscription; TWDM-PON.**

I. Introduction

At present, some network operators have begun to offer 1 Gb/s downstream Internet access to both residential and business customers in the world. Such bandwidth capacity may not be strictly necessary for supporting new services, but customers appreciate such bandwidth as a means to enhance their experience, especially given the ever-increasing number of devices connected at home (laptops, tablets, smartphones, smart TVs, etc). Other services than residential, namely business services or wireless backhaul, require better service levels than conventional basic connections, in particular, low-latency and higher bandwidth guarantees.

The Passive Optical Network (PON) is the technology of choice in the medium and long term due to its speed and cost-effectiveness. Indeed, GPON and EPON are becoming widely spread in the access network, and their next-generations have been recently standardised. In all cases, the Optical Line Terminal (OLT) located at the Central Office (CO) arbitrates access to the shared media thanks to a Dynamic Bandwidth Allocation algorithm which allocates transmission windows to the users in a TDM-based sharing model. In addition, the total bandwidth is shared between all users in a dynamic fashion, allowing the OLT to assign bandwidth only to active users as needed.

It has been observed that only a few number of subscribers are simultaneously active in a number of scenarios [1], [2], [3], [4], [5], thus allowing network designers apply oversubscription models in their capacity plans and leverage statistical multiplexing gains to reduce the cost of deployment. In particular, the work in [3], which is an Adtran white paper available at the FTTH Council website[1], estimates the average traffic generated per household in North America as 60 Kb/s in 2008, which translates into about 1 Mb/s to year 2016 (assuming 50% Cummulative Annual Growth Rate). The authors in [4] collect measurements from seven major ISPs in Japan and estimate that the peak residential traffic is about 250 Gb/s for 14.5 Million residential users for 2004 data, which accounts to an average of 17.2 Kb/s per user; again translating this number to year 2016 (assuming 50% CAGR) gives: $17.2 \times 1.5^{12} = 2.23$ Mb/s per user on average.

The question is to what extent residential and business users can be mixed on the same PON while keeping their respective service requirements. This letter extends the model of [1], [2] to a more generic case whereby two types of Internet connections are demanded by customers (basic and premium for residential and business users) and further evaluates the limits of capacity planning with oversubscription in Next-Generation PON scenarios. We show that only Next-Generation PON networks can reach a large number of users (split ratio 1:256) with acceptable service levels.

II. Quick overview of TDM-PON technologies

	GPON	XG-PON	TWDM-PON
ITU-T Standard	G.984	G.987	G.989
DL/UL Rate	2.5/1.25	10/2.5	40/10
(Gb/s)		10/10	80/20
Max. Split	1:64	1:128	1:256

TABLE I
SUMMARY OF FEATURES FOR ITU-T TDM-BASED PON TECHNOLOGIES

Table I shows a brief summary of the main features of the ITU-T standards for Passive Optical Networks: GPON, XG-PON and TWDM-PON. Both GPON and XG-PON networks use one wavelength for downstream and another one for upstream, shared by the ONUs in a TDM-fashion. On the contrary, TWDM-PON comprises a stacking of four or eight XG-PONs on different wavelengths (4x10G/2.5G, or 8x10G/2.5G). It is also worth noting that both GPON and XG-PON may coexist on the same ODN, but the wavelength plans of TWDM-PON does not allow GPON or XG-PON users on the same ODN.

III. Capacity planning with oversubscription

A. All users with the same profile

Let N refer to the maximum number of users physically attached to the same PON branch; typical values of N follow powers of two, typically: $N \in \{8, 16, 32, 64, \ldots\}$ depending on the optical splitters. Next, we consider users as two-state Bernoulli processes, i.e. active with probability q or idle with probability $1-q$; all users are uncorrelated and have the same behavior. As previously stated, the value of q is very small for residential users.

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[1]Available in Jan 2016 at: http://www.ftthcouncil.org/d/do/20

Next, let X refer to the random variable that considers the number of active users at a given random time. Clearly, $0 \leq X \leq N$. Essentially, X follows a Binomial distribution, $X \sim B(N, q)$, whose Probability Density Function (PDF) follows:

$$P(X = j) = \binom{N}{j} q^j (1 - q)^{N-j}, \quad j = 0, 1, \ldots, N \qquad (1)$$

Concerning bandwidth, let B denote the rate observed per individual user in the PON branch, as follows:

$$B = \min\left\{\frac{C}{X}, B_{peak}\right\} \qquad (2)$$

where C is the total offered bandwidth capacity of each NG-PON technology (see Table I), and B_{peak} is the maximum (peak) bandwidth rate provided to the users. B is a discrete random variable, which depends on the number of active users: the smaller the value of X the more bandwidth rate experienced per active user (upper bounded by B_{peak}). On the contrary, when all users are active ($X = N$), all users experience the minimum guaranteed bandwidth rate $\frac{C}{N}$. In light of this, the random variables B and X are related as follows:

$$P\left(B \geq \frac{C}{k}\right) = P(X \leq k) = \sum_{j=0}^{k} \binom{N}{j} q^j (1 - q)^{N-j} \qquad (3)$$

meaning that, when k users are active, the capacity C is equally shared among them. In other words, a minimum of C/k bandwidth is available for users as long as no more than k users are active.

Both users and operators are interested in two metrics regarding bandwidth: average value $E(B)$ perceived by active users and percentage of time β whereby a certain peak bandwidth B_{peak} is guaranteed. The former follows:

$$\begin{aligned} E(B) &= \sum_{j=0}^{N} \min\left\{\frac{C}{j}, B_{peak}\right\} P(X = j) \\ &= \sum_{j=0}^{N_{act}^{(max)}} B_{peak} \binom{N}{j} q^j (1 - q)^{N-j} \\ &+ \sum_{j=N_{act}^{(max)}}^{N} \frac{C}{j} \binom{N}{j} q^j (1 - q)^{N-j} \end{aligned} \qquad (4)$$

where the term $N_{act}^{(max)}$ is the maximum number of active users that allows them receive B_{peak} fixed, i.e.:

$$N_{act}^{(max)} = \left\lfloor \frac{C}{B_{peak}} \right\rfloor \qquad (5)$$

On the other hand, the percentage of time β whereby bandwidth B_{peak} is guaranteed to an active user can be obtained as follows:

$$\beta = P\left(X \leq \left\lfloor \frac{C}{B_{peak}} \right\rfloor\right) \qquad (6)$$

In other words, the percentage of time β is guaranteed when no more than $\frac{C}{B_{peak}}$ users are simultaneously active.

In general, it is very unlikely to have many active users when q is sufficiently small. This allows network operators to leverage statistical multiplexing gains and plan their networks with oversubscription.

B. Numerical example and design criteria

Consider a network operator willing to deploy a GPON network ($C_{DL}^{(GPON)} = 2.5$ Gb/s downlink) whereby users are guaranteed $B_{peak} = 1$ Gb/s during at least 20% of the time (i.e. β_{min}). In addition, this deployment is intended for residential users who have shown $q = 0.15$ (i.e. 15% average activity) in the past.

First of all, users are guaranteed $B_{peak} = 1$ Gb/s at least only if at most two of them are active. In a deployment with $N = 16$ users (1:16 split), this occurs with probability 0.56 which is greater than $\beta_{min} = 0.2$. However, if the operator decides to go for a deployment with $N = 32$ users, then $P(X \leq 2) = 0.12$ when $X \sim B(32, 0.15)$. Thus, the operator should plan its GPON network with a split ratio of at most 1:16. In such a case, the average bandwidth perceived by users equals to $E(B) = 870$ Mb/s as it follows from eq. (4). It is also worth noting that all $N = 16$ users are active with probability $q^N = 0.15^{16} \sim 10^{-14}$ which is unlikely; in such an unlikely case, each user perceives an equal share of the total 2.5 Gb/s bandwidth, i.e. 156 Mb/s which is the worst case value possible.

Following this methodology, Table II shows the values of β and $E(B)$ for GPON, XG-PON and TWDM-PON with different split ratios under the assumptions of users with low periods of activity (i.e. residential-like $q = 0.15$) and users with high-periods of activity (i.e. business-like $q = 0.5$).

When $q = 15\%$, XG-PON significantly improves the results of GPON providing B_{peak} during a large percentage of time, allowing split ratios of up to 1:64. TWDM-PON provides at least 1 Gb/s during nearly 100% of the time for split ratios up to $1:128$ and may even reach $1:256$ with good performance ($\beta = 63\%$ and $E(B) = 929$ Mb/s). When large user activity periods are expected, for instance $q = 50\%$, only TWDM-PON can reach up to 64 users with a minimum of B_{peak} during 20% of the time.

In summary, the split ratio dimensioning problem for a given PON technology reduces to finding the largest value of N that satisfies:

$$P\left(X \leq \left\lfloor \frac{C}{B_{peak}} \right\rfloor\right) \geq \beta_{min} \qquad (7)$$

where $X \sim B(N, q)$. Such largest value of N must be rounded to the next smallest split ratio possible, i.e. $\{2, 4, 8, 16, 32, 64, 128\}$.

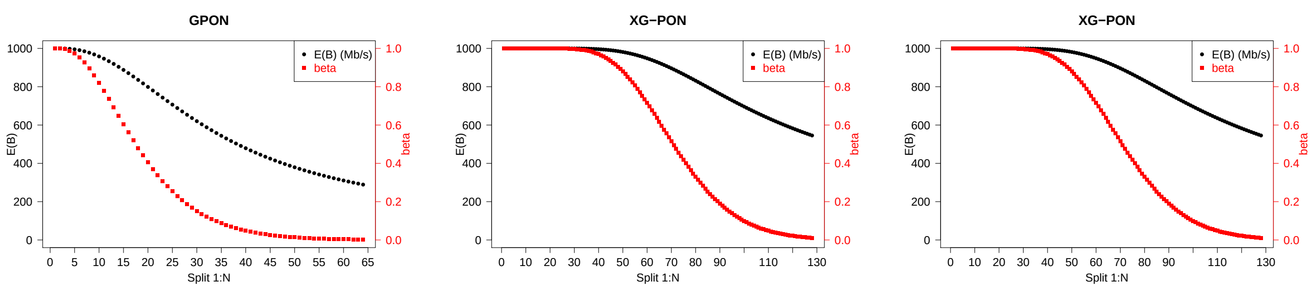
IV. Analysis with two profiles: basic and premium users

The previous analysis has considered that all users show the same activity q and have the same β_{min} requirement. Now, consider the case with two different types of users, namely, (1) basic users that show low values of q and demand B_{peak} only during a low value of β, and (2) premium users that show large values of q and require B_{peak} guaranteed during $\beta_{min} = 100\%$ of the time. As shown in [6], some operators

	Split						
	1:4	1:8	1:16	1:32	1:64	1:128	1:256
	q = 15%						
GPON	998 Mb/s	977 Mb/s	870 Mb/s	588 Mb/s	289 Mb/s	-	-
	99%	89%	56%	12%	~%	-	-
XG-PON	1000 Mb/s	1000 Mb/s	1000 Mb/s	999 Mb/s	929 Mb/s	545 Mb/s	-
	~100%	~100%	99%	99%	63%	1%	-
40G-TWDM	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	999 Mb/s	929 Mb/s
	~100%	~100%	~100%	~100%	99%	99%	63%
80G-TWDM	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	999 Mb/s
	~100%	~100%	~100%	~100%	~100%	99%	99%
	q = 50%						
GPON	934 Mb/s	665 Mb/s	337 Mb/s	161 Mb/s	79 Mb/s	-	-
	68%	14%	~0%	~0%	~0%	-	-
XG-PON	1000 Mb/s	1000 Mb/s	987 Mb/s	645 Mb/s	317 Mb/s	157 Mb/s	-
	~100%	~100%	89%	2%	~0%	~0%	-
40G-TWDM	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	987 Mb/s	645 Mb/s	317 Mb/s
	~100%	~100%	~100%	~100%	89%	2%	~0%
80G-TWDM	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	1000 Mb/s	987 Mb/s	645 Mb/s
	~100%	~100%	~100%	~100%	~100%	89%	2%

TABLE II

BANDWIDTH COMPARISON BETWEEN THE MAIN PON TECHNOLOGIES: $E(B)$ AND β WHEN B_{peak} = 1 GB/S



(a) (b) (c)

Fig. 1. $E(B)$ and β for the three PON technologies (GPON, XG-PON and 40G-TWDM)

have observed that residential and business users exhibit different patterns, i.e. non-overlapping peak times (the busiest hour of business and residential users occurs in the morning and evening respectively). For simplicity and brevity, we consider that both residential and business users are uncorrelated and may be active during the same period of time as a worst-case scenario.

In particular, we shall consider B_{peak} = 1 Gb/s, $q_p = 0.5$ for premium users and a strict requirement of 1 Gb/s available during $\beta_{min}^{(p)} = 100\%$ of the time, whereas basic users show $q_b = 0.15$ and only require 1 Gb/s during $\beta_{min}^{(b)} = 20\%$ of the time.

Let $0 \leq X_b \leq N_b$ and $0 \leq X_p \leq N_p$ denote the number of basic and premium active users, and let $P(X_b \leq j, X_p \leq k)$ refer to the joint probability distribution function of active basic and premium users. N_b and N_p are the maximum number of active basic and premium users in a PON. These numbers must be dimensioned based on the values of $\beta_{min}^{(b)}, q_b$ and $\beta_{min}^{(p)}, q_p$, and also taking into account that $N_p + N_b = N$ must be rounded to a power of two.

In the case of GPON (2.5 Gb/s downlink), when B_{peak} = 1 Gb/s, at most two premium users are possible in a PON since it would not be possible to guarantee 1 Gb/s to three users during 100% of the time. Thus, the maximum number of premium users N_p is limited to:

$$N_p = \left\lfloor \frac{C}{B_{peak}} \right\rfloor \tag{8}$$

Next, the goal is to find the largest value of basic users N_b for sharing the remaining capacity, whereby $\beta_{min}^{(b)}$ is guaranteed. In this example, if $N_p = 2$ premium users are present in the PON, the average capacity they spend follows $N_p q_p B_{peak}$ since they are not always active (only during q_p per user), leaving the rest of capacity for basic users.

Essentially, for every pair (N_p, N_b) where $N_p + N_b$ is a power of two, we need to compute the percentage of time

$E(B_b),\ \beta_b$ / n_p	GPON 1:8	GPON 1:16	GPON 1:32	XG-PON 1:16 / 40G-TWDM 1:64 / 80G-TWDM 1:128	XG-PON 1:32 / 40G-TWDM 1:128 / 80G-TWDM 1:256	XG-PON 1:64 / 40G-TWDM 1:256 / 80G-TWDM 1:512*
$n_p = 0$	977, 89%	870, 56%	588, 12%	1000, ~100%	999, 99%	929, 63%
$n_p = 1$	947, 82%	795, 46%	499, 9%	1000, ~100%	998, 99%	913, 59%
$n_p = 2$	870, 72%	678, 36%	395, 6%	1000, ~100%	998, 98%	895, 54%
$n_p = 3$	-	-	-	1000, ~100%	997, 98%	875, 49%
$n_p = 5$	-	-	-	1000, ~100%	992, 96%	825, 40%
$n_p = 7$	-	-	-	999, ~100%	982, 92%	762, 31%
$n_p = 9$	-	-	-	998, ~100%	959, 86%	686, 23%
$n_p = 10$	-	-	-	994, 99%	941, 83%	643, 19%

TABLE III
BANDWIDTH COMPARISON FOR DIFFERENT BUSINESS/RESIDENTIAL CONFIGURATIONS IN GPON, XG-PON AND TWDM-PON.

where the basic users receive B_{peak}:

$$P\left(X_b \leq \left\lfloor \frac{C - B_{peak}X_p}{B_{peak}} \right\rfloor\right) = \sum_{k=0}^{N_p} P\left(X_b \leq \left\lfloor \frac{C - B_{peak}k}{B_{peak}} \right\rfloor \middle| X_p = k\right) P(X_p = k) \quad (9)$$

and the average capacity $E(B_b)$ experienced by them:

$$E(B_b) = \sum_{k=0}^{N_p} E(B_b|X_p = k)P(X_p = k), \quad X_p \sim B(N_p, q_p) \quad (10)$$

Table III shows the average bandwidth $E(B_b)$ and percentage of time where B_{peak} is provided to basic users, for different values of $n_p = 0, \ldots, \left\lfloor \frac{C}{B_{peak}} \right\rfloor$.

In the case of TWDM-PON, the calculus can be reused from the XG-PON results, since TWDM-PON is in fact a stacking of four or eight XG-PONs on different wavelengths. For instance, 40G-TWDM-PON with split 1:128 is essentially a stacking of four XG-PONs on four different wavelenghts, thus allowing 32 users per lambda (4x32). 80G-TWDM-PON with split 1:256 gives the same numbers as XG-PON (1:32). Thus, at most ten business users can be located on the same wavelength of a TWDM-PON (i.e. 40 business total for 40G-TWDM-PON and 80 business users for 80G-TWDM-PON). It is finally worth remarking that splits 1:512 are not yet supported by the ITU-T G.989 standard.

V. Discussion and Future Work

Essentially, GPON provides good results for splits 1:8 and 1:16, but cannot meet the requirements for 1:32 and beyond. XG-PON is recommended for splits 1:32 and even 1:64 with a moderate number of premium users. 40G-TWDM-PON shows the same performance, but for splits 1:128 and 1:256, but 80G-TWDM-PON con go beyond these numbers and even reach 512 users. Clearly, TWDM-PONs provide the best performance results and highest number of connected users. Furthermore, TWDM-PONs are cost-effective technologies in dense urban areas, where the average distance between the ONUs and the OLT is small and the Optical Distribution Network (ODN) can be shared by a large number of customers, not that much in rural areas (see [7] for further details).

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